SYNTHESIS OF A NETWORK FOR A PRESCRIBED TIME FUNCTION*

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(Received for publication, July 51 1954)

ABSTRACT. The paper discusses the problem of the synthesis of a linear network for a specified time response. Two methods, the moment generating function method and the time series method, are presented. In the former, the generating function of the transfer immitance is found on the basis of moment approximation, and in the latter, the free modes of the system are determined from the spectrum or from the regressive equation of the time series of the waveform. It is shown that the methods presented give physically realisable network structures under quite general conditions. Three examples are considered to illustrate the procedure.

INTRODUCTION

A linear network can be specified either by its frequency response or by its waveform response to any standard input. In the past one was generally concerned with the frequency response of a network, and as such, one's task was to design the network for a prescribed frequency response. Modern communication techniques, e.g. radar, servo-mechanisms, pulse modulation system etc., are, however, concerned more directly with the waveform response. Further, one is also required to generate special waveforms, for example, the normal error pulse shape which is known to be the best pulse shape for detection of weak signals. Such demands have caused the attention to be directed to the problem of systhesis of networks with a prescribed waveform or time response.

The problem of synthesizing a network having a specified time response involves, firstly, the choice of a proper basis of approximation and, secondly, obtaining the approximate network function in a physically realisable form. For the former some criterion of goodness of approximation has to be applied and those generally used are the least squre deviation and the Tschobycheff approximation.

Several methods of synthesizing a network for a specified time response are reported in the literature on the subject. Much work, however, still remains to be done before a general solution of the problem is obtained. The present paper aims at making some contributions to the solution of the problem by suggesting extensions of and possible alternatives to the existing methods of attack. The methods that will be described here are, (a) the

* Communicated by Prof. S. K. Mitra.

moment generating fnnction method and (b) the time series method. In the first method one identifies the coefficients of the power series of the transfer immitance with the moments of the time function. The power series is assumed to be a recurrent one and its generating function is then found. A simple algebraic procedure is then shown to give the network function. The application of the method is, however, restricted to cases where the time function is monotonic. In the second method the free modes of the network are determined from the characteristics of the time series. For this purpose one forms the periodogram or the correlogram, or constructs the autoregression equation. Then, with the estimates of the location of the poles of the network, one selects the zeros for least square approximation to the given time function.

Before going into the suggested methods we give a brief summary of the principal works in the field.

SUMMARY OF PREVIOUS WORK

As usual, we define the transfer immitance $g(p)$ as

$$g(p) = \frac{Lf(t)}{LS(t)} \qquad (1)$$

where $LS(t)$ and $Lf(t)$ are Laplace transforms of the input $S(t)$ and output $f(t)$ respectively. In what follows $S(t)$, unless otherwise stated, will deno'e Dirac's δ function, so that $f(t)$ becomes the weighting function $W(t)$ specifying $g(p)$. The practical problem now is to find a physically realisable network function $\Phi(p)$ which approximates so closely to $g(p)$ as to render

$$U = \int_0^\infty \{L^{-1}(\Phi(p) - g(p)\}^2 dt \qquad \ldots(2)$$

a minimum, L^{-1} meaning the inverse Laplace transform.

Now, in order that $\Phi(p)$ be the transfer immitance of a finite network with lumped constants it must be of the form

$$\Phi(p) = \frac{\sum^m a_r p^r}{\sum^n b_r p^r} = \frac{N(p)}{D(p)} \qquad \ldots \quad (3)$$

where $N(p)$ and $D(p)$ are rational polynomials in p satisfying the following conditions:

(a) the coefficients of $N(p)$ and $D(p)$ are all real,

(b) the zeros of $D(p)$ cannot lie in the right half p-plane,

(c) degree of $N(p)$ cannot be higher than that of $D(p)$.

Airgrain and Williams (1949a) have suggested a method resting on the minimisation of the integral (2). Writing $\Phi(p) = \Sigma \frac{A_S}{p + \alpha_S}$ one forms the system of equations

$$\frac{\partial U}{\partial \alpha_S} = 0, \quad \frac{\partial U}{\partial A_S} = 0 \quad (S = 1, 2 \ldots n)$$

solutions of which give α_S and A_S. Airgrain and Williams, (1949*a*) have also extended the method by employing the Laguerre series expansion of any time function.

Thomson (1952) has adopted the method of moments for finding the approximate network function. He takes a special Hurwitz polynomial as the denominator and then finds the numerator polynomial.

Another method (Nadler, 1949), useful for realising the input impedance function employs the continued fraction expansion of the Poisson-Stieltjes integral,

$$g(p)=pC+\int^{\infty} \frac{d\psi(x)}{p^2+x}, \text{ where } d\psi(x)=Re\frac{g(i\sqrt{x})}{\pi\sqrt{x}}\,dx,$$

and

$$C=\lim_{p\to\infty}\frac{g(p)}{p}.$$

The Fourier series representation of $f(t)$ may also be used for obtaining the transfer immitance. More generally, the output time function may be represented by means of any orthogonal complete set having rational Fourier transform, so that

$$f(t)=\Sigma\beta_n\psi_n(t) \text{ and } g(p)=\Sigma\beta_n\chi_n(p) \qquad \dots \quad (3a)$$

where $\chi_n(p)$ is the Fourier transform of $\psi_n(t)$. The coefficients β_n are obtainable from $\beta_n=\dfrac{\int f(t)\psi_n(t)dt}{\int[\psi_n(t)]^2dt}$.

It should be noted that Fourier series representation implies choice of commensurate complex poles while Laguerre series representation implies selection of *n*th order real pole.

THE MOMENT GENERATING FUNCTION METHOD

If in the fundamental relation $g(p)=\int_0^{\infty} e^{-pt}f(t)dt$, $f(t)$ is normalised, that is, if $\int_0^{\infty} f(t)dt=1$, then on expanding e^{-pt} we have

$$g(p)=\int_0^{\infty} f(t)dt-p\int_0^{\infty} tf(t)dt+\dots+\frac{(-p)^r}{(r)!}\int_0^{\infty} t^rf(t)dt+\dots$$

$$=1-\mu_1'p+\mu_2'\frac{p^2}{2!}+\dots+\frac{(-p)^r}{r!}\mu_r'+\dots \qquad \dots \quad (4)$$

where μ_r' denotes the *r*th order moment about the origin. Differentiating $g(p)$ r times one obtains

$$\frac{\partial^r}{\partial p^r}\,g(p)=\int (-t)^rf(t)\,e^{-pt}dt. \qquad \dots \quad (4a)$$

Thus the coefficients of the power series expansion of $g(p)$ are obtainable from the transform of $(-t)^r f(t)$.

The moments can be related to the real and imaginary parts, $A(\omega)$ and $B(\omega)$ respectively, of the transfer function $g(j\omega)$. From

$$\mu'_r = (-1)^r \left[\frac{\partial^r}{\partial p^r} g(p) \right]_{p=0}$$

we note

$$A(0) = 1,\ B'(0) = \mu_1',\ A''(0) = \mu_2',\ \ldots\ A^{2n}(0) = (-1)^n \mu_{2n}' \qquad \ldots (5)$$

The moments can also be related to the delay time and rise time thus : Delay time can be defined as

$$t_d = \int_0^\infty t f(t)\,dt = \mu_1' \qquad \ldots (6a)$$

and the rise time as $t_r = \left[2\pi \int_0^\infty (t - t_d)^2 f(t)\,dt \right]^{\frac{1}{2}} = \left[2\pi\mu_2 \right]^{\frac{1}{2}}$ .. (6b)

It will be seen that $g(p)$ determines the moments (when these exist) and hence the distribution function. It should, however, be observed that a set of moments determines the distribution uniquely only under certain restrictions. A criterion of unique determination is that $\overline{\text{Lim}}\ \frac{\mu_n^{1/n}}{n!}$ is finite. This criterion is derived from the condition of convergence of the power series of $g(p)$. It is worth noting here that two distributions having identical moments up to the nth order are equal in the sense of least square approximation. It is also to be noted that the method of moments is applicable only if the time response is monotonic.

As already stated, $\Phi(p)$, in order to be realisable in the form of a network, must be of the form $\Phi(p) = \sum_0^m a_r p^r / \sum_0^n b_r p^r$. Here, $n > m$, for $n = m$ implies an impulse at $t = 0$ and such cases are outside our purview. Also, it has been mentioned that for realising $\Phi(p)$ one may choose a rational Hurwitz polynomial for the denominator and then adjust the numerator to approximate to $g(p)$. The alternative suggested here is to consider instead the power series of $g(p)$ as a recurring series of order n and then find the coefficients of the generating function, i.e., the denominator polynomial $D(p)$, from the set of recurrent relations

$$\frac{b_n \mu'_k}{k!} - \frac{b_{n-1}\mu'_{k+1}}{(k+1)!} + \quad \ldots \quad + (-1)^n \frac{\mu'_{k+n}}{(k+n)!} \qquad \ldots (7)$$

Equation (7) will be recognised as an nth order difference equation with constant coefficients. If the roots of the characteristic equation

$$x^n - b_1 x^{n-1} + \quad \ldots \quad + (-1)^n b_n = 0 \qquad \ldots (8)$$

are $C_1, C_2, \ldots C_n$, then $\frac{\mu_r}{r!} = A_1 C_1^r + A_2 C_2^r + \ldots + A_n C_n^r$. An evident constraint on the roots is that $\frac{\mu_r'}{r!} \to 0$ as $r \to \infty$. Hence $0 < | C_K | < 1$.

Now the solution of $D(p)$ from the set of relations (7) and $D(p) = \sum_0^n b_r p^r$ can be presented in the form

$$D(p) = \frac{C}{\Delta_{n-1}} \begin{vmatrix} 1 & p & p^2 \ldots p^n \\ \nu_1 & \nu_2 & \ldots \nu_n \\ \nu_{n-1} & \nu_n & \ldots \nu_{2n-1} \end{vmatrix}, \text{ where } \nu_n = (-1)^n \frac{\mu_n'}{n!} \tag{9}$$

and $\Delta_n = \begin{vmatrix} \nu_0 & \cdots & \nu_n \\ \vdots & \ddots & \\ \nu_n & & \nu_{2n} \end{vmatrix}$ and C is a constant.

The numerator polynomial will then be

$$N(p) = 1 + (\mu_1' - b_1)p + \left(\frac{\mu_2'}{2} - b_1\mu_1' + b_2\right)p^2 + \ldots$$

$$+ (-p)^{n-1}\left[\frac{\mu'_{n-1}}{(n-1)!} - \frac{b_1 \mu'_{n-2}}{(n-2)!} + \ldots (-1)^n b_{n-1}\right] \tag{10}$$

Equations (9) and (10) determine the network function completely. We may now write $\Phi(p)$ as

$$\Phi(p) = \frac{N(p)}{D(p)} \text{ or as } \sum \frac{A_K}{p + \gamma_K}$$

or as

$$\frac{1}{\alpha_0 p + \alpha_1} + \frac{1}{\alpha_2 p + \alpha_3} + \frac{1}{\alpha_4 p + \alpha_5} + \tag{11a}$$

From the theory of linear simultaneous equations, it is known that for the parameters $b_1, b_2, \ldots\ldots, b_n$ to be independent, it is necessary that the rank of the matrix of the system be equal to the order. This sets the upper limit to the order of the polynomial. We have now to ensure that the b_r's are all positive and that $D(p)$ is Hurwitz. It should be observed that if the power series of $g(p)$ converges, so will the continued fraction and the rational fraction associated with it; further that for boundedness of $f(t)$, it is required that $g(p)$, and hence $D(p)$, be regular when $Re\, p > 0$. It is easy to prove that in the continued fraction

$$\Phi(p) = \frac{1}{\alpha_0 p + \beta_1} + \frac{1}{\alpha_1 p + \beta_2} + \frac{1}{\alpha_2 p + \beta_2} \tag{11b}$$

if α_r's and β_r's are all positive, the roots of $D(p)$ are simple, real and negative. For this we consider the sequence $q_{n+1}, q_n, \cdots q_1, q_0$ as Sturm functions where $q_{r+1} = (\alpha_r p + \beta_r)q_r + q_{r-1}$, where p_r/q_r is the r th convergent.

It is evident that none of the Sturm functions can pass through zero in the interval o to ∞ on the axis of reals and that n changes of sign will be lost in the sequence of Sturm functions as p passes from ∞ to $-\infty$.

It can easily be demonstrated that if $dF(t)$ has not less than $n+1$ points of increase one must have $\Delta_1 > 0$, $\Delta_2 > 0$, ... $\Delta_n > 0$, and conversely if the inequalities are satisfied, $dF(t)$ has at least $n+1$ points of increase, where

$$\Delta = \begin{vmatrix} \mu_0' & \cdots & \mu_n' \\ \vdots & \ddots & \\ \mu_n' & & \mu_{2n}' \end{vmatrix}, \quad \mu_r' \text{ meaning the } r\text{th moment.}$$

For this we easily see that the quadratic form

$$\theta = \int_{-\infty}^{\infty} (U_0 + U_1 t + \ldots + U_n t^n)^2 dF(t) = \Sigma \mu'_{i+K} U_i U_K$$

is definitely positive, for, by hypothesis $dF(t)$ has at least $(n+1)$ points of increase and at least one of these must be different from all the zeros of

$$U_0 + U_1 t + \ldots\ldots + U_n t^n$$

so that the integral is always positive so long as U_i's are not all zero. This positive definite character ensures the determination of $dF(t)$ with a set of moments μ_r'.

This result may profitably be used in realising a minimum phase shift type transfer function when its real part is non-negative. Consider the Poisson line integral

$$g(p) = \frac{2p}{\pi p} \cdot \int \frac{Re g(j\omega)}{p^2 + \omega^2} d\omega = \frac{2p}{\pi} \int_0^{\infty} \frac{Re\, g(j\omega)}{p^2} d\omega \left[1 - \frac{\omega^2}{p^2} + \frac{\omega^4}{p^4} - \ldots \right]$$

$$= \frac{2}{\pi p} \cdot \left[m_0 - \frac{m_2}{p_2} + \frac{m_4}{p^4} \ldots\ldots \right]$$

where ms is the sth moment and $Re g(j\omega)$ is the real part of $g(j\omega)$. Stipulating that on expanding $\int^{\infty} \frac{Re\, g(j\omega)}{p^2 + \omega^2} d\omega - \frac{N(p^2)}{D(p^2)}$ into a power series of $\frac{1}{p^2}$, terms involving $\frac{1}{p^2}$ to $\frac{1}{p^{4n}}$ are absent, we obtain

$$\int^{a} \theta(\omega) D(\omega)\, Re\, g(j\omega) d\omega = 0 \qquad (12b)$$

for an arbitrary polynomial θ of degree S, $S \leq n$. It is easy to show that the roots of $N(p^2)$ and $D(p^2)$ are real, simple and contained within the interval o to $-\infty$. Equations (12) thus completely determine the network function.

We may add in conclusion that it sometimes saves labour and yields more accurate result if orthogonal series expansion of $g(p)$ instead of the actual power series is taken. This procedure is particularly preferable when the form of the transfer immitance is readily obtainable. As the orthogonal series we may take the Laguerre series in the interval o to ∞, or the Tschebycheff or the Legendre series in the reduced interval -1 to $+1$.

TIME SERIES METHOD

A time series of a wave form $f(t)$ is a sequence of its values at equal intervals of time, i.e., the sequence

$$(f_0, 0),\ (f_1, \alpha),\ \ldots\ldots (f_K,\ K\alpha)$$

where f_r is the value of $f(t)$ at $r\alpha$, α being the spacing. It is well known that if ω_0 is the highest frequency of interest in the spectrum of a band-limited function $f(t)$ then it can be represented as

$$f(t) = \sum_0^{2\omega_0 T} f\left(\frac{r}{2\omega_0}\right) \frac{\sin \pi(2\omega_0 t - r)}{\pi(2\omega_0 t - r)}\ ,\ T \text{ being the total interval} \tag{13}$$

The importance of the relation is that it enables one to specify a continuous function of time in terms of its values at intervals of $1/2\omega_0$ apart. For example, the response of a low pass filter with frequency response, uniform up to a frequency f_c and zero for all other frequencies can be represented by a time series of interval $1/2f_c$.

The time series, it will be understood, describes completely the characteristic of a linear network. If the response of the linear network to an input $S(t)$ be $f(t)$, then $f(t) = \int_0^\infty S(\tau) W(t-\tau) d\tau$ where $W(t)$ is the weighting function. The time series of $f(t)$ and $S(t)$ can be used to determine $W(t)$ completely. The problem now is to realise the transform of $W(t)$. For this, we shall describe two alternative methods for finding out the poles of the approximate network : (*a*) the spectrum method and (*b*) the regressive equation method.

(*a*) ***Spectrum Method.***

The relation

$$g(p) = \int^\infty W(t) e^{-pt} dt = \alpha \sum W(r\alpha) e^{-pr\alpha} \tag{14}$$

for the transfer function is satisfied by

$$g(p) = \sum_1 \frac{}{p+\alpha_r} + \sum_1^V \frac{b_r \omega_r}{p^2 + \omega_r^2} \cdot \sum_1^Q \frac{}{p^2 + \omega_r^2} \tag{14a}$$

if, at discrete points $t = \tau$,

$$W(\tau) = \Sigma a_r e^{-\alpha_r \tau} + \Sigma b_r \sin \omega_r \tau + \Sigma c_r \cos \omega_r \tau \tag{15}$$

where α_r's denote the real poles and ω_r's the imaginary poles. One has now to find the poles α_r and ω_r and the coefficientf a_r, b_r, c_r. For this one first forms the correlogram or the periodogram or constructs the spectrum by conventional methods. To form the correlogram we first write

$$r_K = \lim_{T\to\infty}\frac{1}{T}\int_0^T f(t)f(t+k)dt \tag{16}$$

and then form $$e(\omega)=\int^{\infty} r_K \cos \omega k dk.$$

To construct the periodogram, we write,

$$A=\int_0^{\infty} f(t)\cos \omega t dt,\ B=\int_0^{\infty} f(t)\sin \omega t dt$$

and then form $$S^2=A^2+B^2 \quad \ldots \quad (17)$$

When the spectrum is such that the number and the locations of the poles cannot be accurately ascertained, reasonable assumptions regarding the same have to be made. One has then to select the unknown a_r, b_r, and c_r for least square approximation to the given time function. For the purpose one forms

$$U=\sum_{r}\left[f(\tau)-\{\Sigma a_r e^{-a_r\tau}+\Sigma b_r \sin \omega_r\tau+\Sigma c_r \cos \omega_r\tau\}\right]^2 \quad \ldots \quad (18)$$

and minimises it with respect to a_r, b_r and c_r, *i. e.* equate $\frac{\partial U}{\partial a_r}$, $\frac{\partial U}{\partial b_r}$ and $\frac{\partial U}{\partial c_r}$ separately to zero. Now $\frac{\partial U}{\partial a_1}=0$ gives

$$a_1\Sigma x_{1m}^2+\Sigma a_K\Sigma x_{Km}x_{1m}+\Sigma b_K \Sigma x_{1m}y_{Km}+\Sigma c_K\Sigma x_{1m}z_{Km}=\Sigma f_m x_1 \tag{18a}$$

where x_{rm}, y_{rm}, z_{rm} and f_m denote the values respectively of $e^{-a_r\tau}$, $\sin\omega_r\tau$, $\cos \omega_r\tau$, $f(\tau)$ at the nth sampling point.

From the set of linear equations ($18a$) one solves a_r, b_r and c_r. $g(p)$ is then in the network realisable form ($14a$).

It is necessary to note that only if the time function is the response of a linear network with finite elements it is possible to find out α_r and ω_r with reasonable accuracy from the correlogram or the periodogram. In the general case of an arbitrary time function, the representation ($3a$), in terms of a complete orthogonal set, has the advantage over the representation ($14a$) that the coefficients β_n are more easily found.

(*b*) *Regressive Equation Method.* The time response of a linear system is known to possess the useful property, namely, that its time series is linear auto-regressive. The basis of linear auto-regression is the fact that an nth order linear differential equation may be regarded as the limiting case of m linear algebraic equations when in the limit m tends to infinity. If we consider the second order differential equation

$$p\ddot{x} + q\dot{x} + rx = \psi(t)$$

where p, q, r are continuous in the interval $0 \leq t \leq T$, the difference equation from

$$p(t_v)\frac{\Delta^2 x_v}{\Delta t^2} + q(t_v)\frac{\Delta x_v}{\Delta t} + r(t_v)x_v = \psi(t_v),$$

where $\Delta x_v = x_{v+1} - x_v$, $\Delta^2 x_v = x_{v+2} - 2x_{v+1} + x_v$; $\left[v = 0, 1, 2, \ldots, \frac{T}{\alpha} - 2 \right.$

and where α is the spacing, will be

$$P_v x_v + Q_v x_{v+1} + R_v x_{v+2} = \psi_v$$

Proceeding similarly the nth order difference equation is

$$P_0 x_v + P_1 x_{v+1} + \qquad + P_n x_{v+n} = \psi_v \qquad v = 0, 1, 2, \ldots, \frac{T}{\alpha} - n$$

In the application we have in mind, the coefficients P_r's will be constants. Then the nth order difference equation for the time series, that is the linear auto-regressive equation with constant coefficients a_1, a_2, ... a_n, is

$$f_{t+n} + a_1 f_{t+n-1} + \ldots\ldots + a_n f_t = 0 \qquad \ldots \quad (19)$$

Here ψ is zero, since we are considering free modes.

There will be $N - n$ such equations. It may be observed that the auto-correlation coefficients are related by the equation

$$r_{m+n} + a_1 r_{m\ n-1} + \ldots\ldots + a_n r_m = 0 \qquad \ldots \quad (20)$$

The solution of (19) is $\qquad f(t) = \sum_1^n A_k Z_k^t \qquad \ldots \quad (21)$

where Z_k are the roots of the equation

$$Z^n + a_1 Z^{n-1} + \ldots\ldots + a_n = 0 \qquad \ldots \quad (22)$$

Now, it will be noted from the differential equation of the output time function $D(p)f(t) = N(p)S(t)$ that the poles, i.e., the roots of $D(p)$ are related to the Z_k by $\gamma_k = \log Z_k$. For boundedness of $f(t)$, $0 < \bmod Z_k < 1$.

The order of the system is to be chosen with reference to the complexity of the network and the rank of the linear equations (19). It is thus seen that the equations (19), through (22), contain all the informations about the locations and the nature of the poles. When the time series shows irregular feature, in setting up the equation (22), we choose instead of (19), the derived equation (20). The transfer function now becomes $g(p) = \sum \frac{A_k}{p + \gamma_K}$. To determine the A_k, we may employ the method of the previous section, i.e., minimise U with respect to A_k, where

$$U = \sum_0^N [f(r) - \sum_1^n A_k Z_k^r]^2.$$

The methods discussed in this section can be used for obtaining wave-form correcting network. If, for example, the impulse response of a networks is $f_1(t)$, and the desired waveform is $f_3(t)$, then, from the convolution integral

$$f_3(t) = \int^{\infty} f_2(u) f_1(t-u) du.$$

one can find by serial division the time series of the corrector, $f_2(t)$. The correcting network can then be designed by the methods of this section.

ILLUSTRATIVE EXAMPLES

(1) To realise the square wave time response defined by $f(t) = k$, $0 < t < T$, and $f(t) = 0$, elsewhere.

Its transform cannot be realised with a finite network, for as is well known, the spectrum of a pulse having steep edges decays as $\frac{1}{p}$ and that of one having corners decays as $\frac{1}{p^2}$. This property is reflected in the fact that the moments of transform form a very slowly convergent sequence. The recurrent series obtained is

$$1 + \frac{x}{2} + \frac{3}{28}x^2 + \frac{x^3}{84} + \frac{x^4}{24 \times 70} \; ; \; (x = pT)$$

$g(p)$ is then

$$\frac{1 + x^2/42}{1 + \frac{x}{2} + \frac{3}{28}x^2 + \frac{x^3}{84} + \frac{x^4}{24 \times 70}}.$$

The network response calculated is observed to improve with the addition of two real poles at 10^6 sec^{-1} and $\frac{1}{3}$ 10^6 sec^{-1} of strength 2. The resultant waveform is shown in figure 1. The network configuration corresponding to $g(p)$ can be found following Cauer.

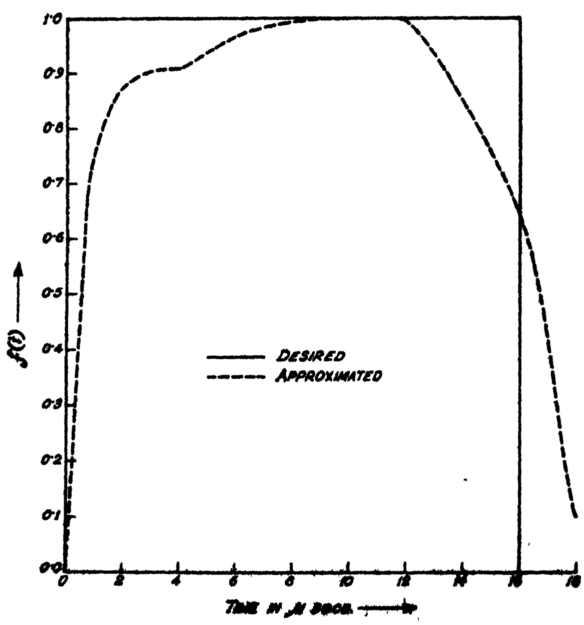

FIG. 1

(2) To synthesize the network for generating a trapezium (figure 2).

The time function has obviously no oscillatory components, i.e., the poles are real ones. Instead of going through the whole procedure, we

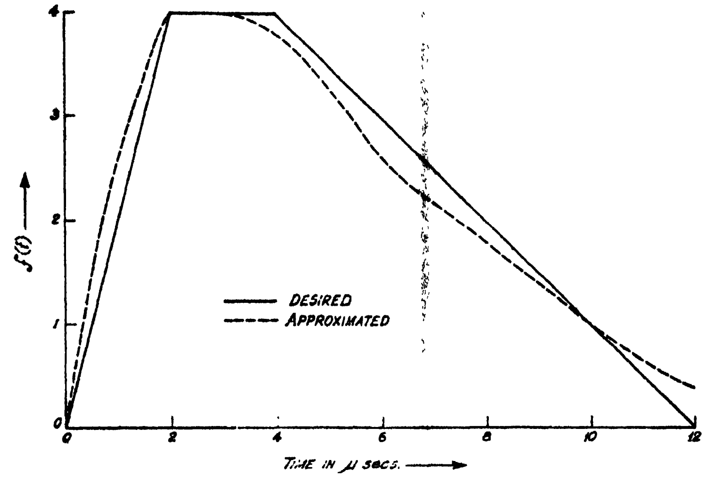

FIG. 2

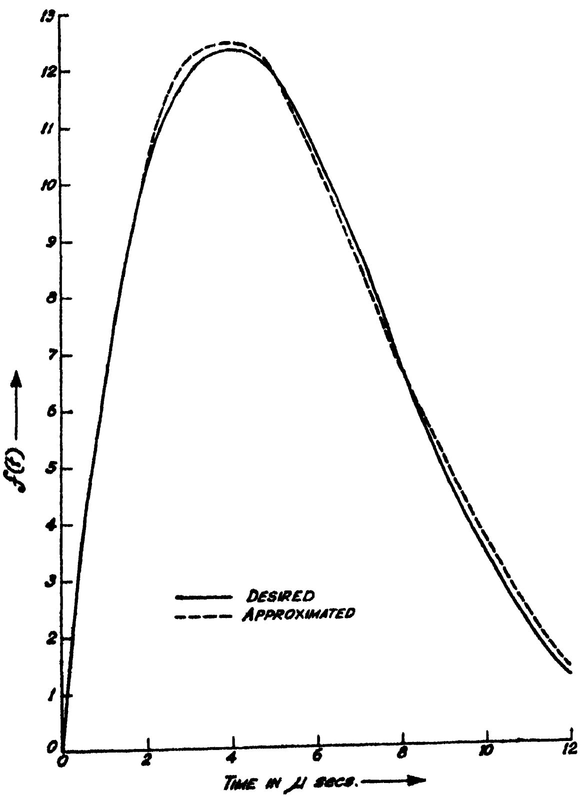

FIG. 3

select, on a preliminary study of the time response, 0.3×10^6, 0.4×10^6, 0.5×10^6, 0.6×10^6, as the locations of the poles.

The transfer function is then

$$\frac{a_1}{p+0.3\times10^6}+\frac{a_2}{p+0.4\times10^6}+\frac{a_3}{p+0.5\times10^6}+\frac{a_4}{p+0.6\times10^6}.$$

The linear algebraic equations referred to in (18*a*) are solved and the constants found are $a_1=38$, $a_2=-52$, $a_3=44$, $a_4=-20$. $g(p)$ is then

$$g(p)=\frac{1.4x^3+7.7x+1.18}{x^4+1.8x^3+1.19x^2+0.342x+.036}.$$

(3) To realise the network for the time response shown in figure 3.

The auto-regression equation of the third order formed is

$$U_{t+3}-2.2145U_{t+2}-1.682\,U_{t+1}+0.440\,U_t=0$$

The roots are $\alpha_1=0.8025$, β_1, $\beta_2=0.706\pm0.31$. The constants A_k are now chosen for best fit. The solution is

$$f(t)=25.8e^{-0.22t}+15.4e^{-0.30t}\sin 0.31t-25.8e^{-0.30t}\cos 0.31t,\ t \text{ being in } \mu \text{ secs.}$$

The network response is found to agree well with the specified time function.

ACKNOWLEDGMENTS

It is a great pleasure to record my indebtedness to Professor S. K. Mitra for his guidance and interest. I am also thankful to Mr. A. K. Chowdhury for his aid and encouragement and to Mr. D. Gupta Sarma for some helpful discussions.

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